A Compressive Sensing Scheme of Frequency Sparse Signals for Mobile and Wearable Platforms

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Abstract. In selected scenarios, sensor data capturing with mobile devices can be separated from the data processing step. In these cases, Compressive Sensing allows a significant reduction of the average sampling rate below the Nyquist rate, if the signal has a sparse frequency representation. This can be motivated in order to increase the energy efficiency of the mobile device and extend its runtime.

Since many signals, especially in the field of motion recognition, are time-dependent, we propose a corresponding general sampling algorithm for time-dependent signals. It even allows a declining average sampling rate if the data acquisition is extended beyond a projected acquisition end.

The presented approach is testified for the purpose of motion recognition by evaluating real acceleration sensor data acquired with the proposed algorithm.

Keywords: Compressive Sensing, Motion Recognition, Data Acquisition, Signal Processing.

1 Introduction

In spite of the tremendous improvements in mobile platform technologies in recent years, the search for techniques to increase the energy efficiency of mobile devices remains an ongoing challenge. This is mainly due to the fact that a mobile device's runtime is directly dependent on a limited source of energy: the battery. For example, the runtime of smartphones has not experienced the same improvements as e.g. the device's processing power.

In many applications, especially in the context of activity recognition or skill assessment, the acquisition of sensor data can be responsible for a substantial share of the consumed energy. For the set of these applications, where it is possible to process and analyse the sensor data offline, we propose to employ the approach of Compressive Sensing[1]. If the acquired signal has a sparse frequency

[1] Also referred to as Compressive Sampling, see [1,2,3,4]

spectrum, it allows a reduction of the sensor activity and thus incites a reduction of total energy consumption on the mobile device.

Applications such as the reconstruction of activities of daily living (ADL), sports activities, or work executed to calculate calorie expenditure, reconstruct the user context, or to assess the quality of an action performed - all this can be done offline, after the actual activity has been conducted. This includes uploading the sensor data either after a certain amount of time or after the end of a sensing phase (indicated by changes in the sampled data below a certain threshold) or at the end of a day, e.g. when returning home from hiking. Many more scenarios are not demanding real-time or online data processing, and thus can benefit from the application of the theory of Compressive Sensing. Examples for the previously mentioned use cases and scenarios are following. The SenseCam [5] is a device hanging around the user's neck. Besides images, it records acceleration sensor data. At the end of a day, this data is downloaded and offline processed. Any subsequent recognition of activities or annotation uses the recorded sensor data. It thus would make sense, especially when the user suffers from dementia, to extend the battery life time of the mobile wearable device, and add a reconstruction step on the desktop computer prior to the activity recognition step. Another example for the offline reconstruction based on multiple, connected wearable embedded and battery powered devices is the research conducted by Laerhoven et al. [6]. They developed the Porcupine sensor platform, an ultra-low power sensing device with communication capabilities. For pure logging tasks, such as long-term activity monitoring or offline activity detection and classification [7], Compressive Sensing can be applied. We argue that, applying the theory of Compressive Sensing, the runtime of such small-scale embedded devices might significantly be increased while allowing the signal to be reconstructed nearly perfectly as we will explain later.

This paper is an extension of our previous work [8] and is structured as follows. We shortly recall the basics of CS in Sec. 2 and introduce our sampling scheme for sparse frequency signals in 3. Our method is evaluated by means of classifying three different motions which arise with the use of a rocker board and are sensed with a mobile device in Sec. 4. We conclude by discussing the strengths and limitations of the presented work in Sec. 5.

2 Compressive Sensing

The well-known Nyquist-Shannon sampling theorem states that a signal, which shall be acquired without loss of information, has to be sampled with a constant frequency that is at least twice the highest frequency present in the signal, [1]. However, an acquisition via this approach can often result in high sampling rates, since the highest frequency within the signal is often not known a priori - or it may not be opportune to exclude a high frequency from detection (e.g. for motion data originating from a specific joint of the human body). Compressive Sensing (CS) has the potential to overcome these problems if the signal that is to be acquired is sparse in the frequency domain. We shortly summarize the basic ideas of CS here in order to lay the theoretical foundation for the presented results.

The concept of CS builds on the notion that many signals of interest have a sparse representation or are compressible. For motion data, for example, the standard representation of a signal is dense. However, since human motion usually follows regular patterns, the Fourier basis typically admits a compressible representation: The important information of a sparse signal is contained in only a few dominant transform coefficients. To reconstruct the entire signal without severe loss of quality, it is sufficient to know these large coefficients together with their respective positions.

The concept of CS [4,9] offers a joint sampling and compression mechanism, which exploits a signal's sparsity to perfectly reconstruct it from a small number of acquired measurements. It allows to acquire a signal, in our example with a mobile device, only partially, and recover the complete data in its sparse representation on a stationary computer. This implies a shift of energy consumption away from the resource-constrained mobile device and onto the energy-unrestricted desktop computer.

Let $f \in \mathbb{R}^n$ be a column vector that represents a discrete n-dimensional nonsparse real-valued signal. We denote its K-sparse representation by $x^0 \in \mathbb{R}^n$, where K-sparse means that only $K < n$ entries of x^0 are nonzero. In the presented application of CS, f denotes the discretized signal in the time domain and x^0 its sparse frequency spectrum. We write the corresponding linear transformation as

$$f = \Psi x^0, \tag{1}$$

where $\Psi \in \mathbb{R}^{n \times n}$ is an orthonormal basis of $\mathbb{R}^n$, called *representation basis*. Furthermore, let $\Phi \in \mathbb{R}^{m \times n}$ be the *sampling basis* that transforms f into the vector $y \in \mathbb{R}^m$ that contains $m < n$ measurements

$$y = \Phi f = \Phi \Psi x^0. \tag{2}$$

We aim to reconstruct f by computing the approximation x^* of x^0, given only the measurements y and exploiting the fact that x^0 is sparse. Informally speaking, we are seeking the sparsest vector x that is compatible with the acquired measurements. Formally, this leads to the following minimization problem

$$\min_x \|x\|_0 \quad \text{subject to} \quad y = \Phi \Psi x, \tag{3}$$

where $\|x\|_0$ is the ℓ_0-pseudo norm of x, i.e. the number of nonzero entries.

Unfortunately, solving (3) is computationally intractable as it is a combinatorial NP-hard problem [10]. Instead, it has been shown in [3,4,11] that under some generic assumptions on the matrix $\Phi\Psi$ it is equivalent to replace the ℓ_0-pseudo norm by the ℓ_1-norm $\|x\|_1 = \sum_i |x(i)|$. Here, $x(i)$ denotes the i-th component of the vector x. This leads to the so called Basis Pursuit

$$\min_x \|x\|_1 \quad \text{subject to} \quad y = \Phi \Psi x. \tag{4}$$

This is a convex optimization problem that can be recast into a linear program, which is solved in polynomial time. The theory of CS says that if the number of measurements m is large enough compared to the sparsity factor K and the measurements are chosen *uniformly at random*, then the solution to equation (4) is exact [4]. This means that the signal is perfectly reconstructed by solving equation (1) with the computed x^*.

We now introduce an important concept, called *coherence*, that influences the amount of required samples. Suppose a signal is sampled in the basis in which it is sparse. In that case, a lot of samples are required for reconstruction, since most of the samples would be zero. Hence, it is intuitively clear that sampling and representation basis have to be *as disjoint as possible*. This is measured by the mutual coherence between Φ and Ψ

$$\mu(\Phi, \Psi) = \sqrt{n} \cdot \max_{i,j} |(\Phi\Psi)(i,j)|, \tag{5}$$

where $X(i,j)$ denotes the (i,j)-entry of the matrix X. The coherence will take a value in $[1, \sqrt{n}]$, cf. [4], and the smaller the value $\mu(\Phi, \Psi)$ the more favorable is the pair of bases [12]. In the presented case, where the sampling and the representation bases are the standard and the Fourier basis respectively, it can be shown indeed that the two bases are maximally incoherent and hence most favorable to a low sampling rate.

The relation between the amount m of required random samples for perfect reconstruction, the coherence $\mu(\Phi, \Psi)$, the sparsity K of x^0, and the dimension n of the signal is provided by the famous formula [13]

$$m \geq C \cdot K \cdot \mu^2(\Phi, \Psi) \cdot \log n, \tag{6}$$

where C is some positive constant. Because of this relation, Compressive Sensing describes a paradigm shift in data acquisition. According to CS, the sampling rate does not depend on the highest frequency within the signal, as stated by the Nyquist-Shannon theorem, but on the signal's sparsity.

3 Sampling a Time-Dependent Signal with Compressive Sensing

As indicated, a key concept of Compressive Sensing is that the m required samples have to be chosen from the n possible samples uniformly at random. That is, from the set $N = \{1, ..., n\}$ of possible sampling points, one of the $\binom{n}{m}$ subsets $M \subset N$, denoting the $|M| = m$ points to be sampled, is chosen randomly with equal probability.

If the samples are to be acquired from a time-dependent signal however, two inconveniences of the uniform selection model become obvious. On the one hand, the duration of the signal acquisition has to be fixed a priori. On the other hand, the random selection has to be made before the start of the signal acquisition. Since the points in time at which the signal is sampled thus have to be stored in memory, this approach can be very memory intensive for long signals. As a main result of this paper, we present a sampling algorithm which overcomes the inconveniences of the uniform probability model and is much more suited for time-dependent sampling. This algorithm is not restricted to applications in motion recognition or to data acquisitions with mobile devices, but can be generally applied in order to acquire time-dependent signals with Compressive Sensing.

In the following we introduce the Bernoulli probability model as in [3]. The set M' of points that are to be sampled is generated by first creating a sequence $I_1, I_2, ..., I_n$ of independent identically distributed random variables with $I_k \in \{0, 1\}$ and $\boldsymbol{P}(I_k = 1) = m/n$, where m and n comply with the constraints given in section 2. The set M' is then created by setting

$$M' := \{k : I_k = 1\}. \tag{7}$$

The mathematical proof that this sampling scheme also leads to a perfect signal recovery can be found in [3,4].

The Bernoulli model already inspires a very practical algorithm for sampling a signal in the time domain. Suppose a signal of a specific duration is sampled at n equidistant points in time according to the Nyquist-Shannon theorem. The Bernoulli model then suggests that the signal can be sampled at each point in time, only with a probability p satisfying the condition

$$p \geq C \cdot K \cdot \mu^2(\Phi, \Psi) \cdot \frac{\log n}{n}. \tag{8}$$

If the dominant frequencies in the signal do not change over time, we can assume an upper bound for the sparsity K which is independent of n. With the coherence $\mu^2(\Phi, \Psi) = 1$ for the bases at hand, equation (8) simplifies to

$$p \geq \text{const} \cdot \frac{\log n}{n}, \tag{9}$$

where the constant value can be estimated experimentally for specific applications. It is remarkable here, that the higher the value n, the lower we may choose the sampling probability p.

Though already providing a lower average sampling rate, this trivial randomized sampling algorithm requires that the mobile computer is reinitialized at each possible sampling point in order to calculate a pseudo-random variable. Since one can expect an even lower energy consumption if the computer is only reinitialized when a sample is actually acquired, we introduce an alternative sampling algorithm in the following. It uses only *one* pseudo-random variable to determine which of the future possible sampling points has to be sampled *next*.

Let L_k be the event that the k-th point of the future potential sampling points is the next one sampled, if randomly determined with the Bernoulli model. The probability of L_k is then given by

$$\boldsymbol{P}(L_k) = p(1-p)^{k-1}. \tag{10}$$

A summation of these probabilities for all $k \in \mathbb{N}$ results in a geometric series which converges to $\sum_{k=1}^{\infty} p(1-p)^{k-1} = 1$.

Since it suffices to randomly determine k, the index of the next sampling point, in accordance with the probabilities of L_k, the alternative algorithm can be stated as follows. At the beginning of the signal acquisition and after each sampled point, the mobile device creates a random variable $X \in [0, 1)$ with uniform distribution and then chooses the next sampling point following the scheme

$$k = \begin{cases} 1, & \text{if } 0 \le X < p \\ 2, & \text{if } p \le X < p(1-p) \\ & \vdots \\ r, & \text{if } p(1-p)^{r-2} \le X < p(1-p)^{r-1} \\ & \vdots \end{cases} \tag{11}$$

which is also illustrated in Figure 1. Once k is determined the mobile computer

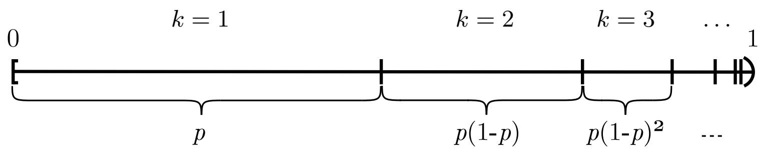

Fig. 1. Illustration of the case differentiation of the presented algorithm

may be put to sleep for the appropriate time frame. The proposed algorithm thus reduces the number of points at which the signal is sampled and at which the mobile computer is reinitialized, both by the factor of p.

The proposed sampling algorithm, together with the probability p as explained above, has two very convenient properties: Firstly, in contrast to the uniform probability model, it is possible to increase the length of the acquired signal from the value n to a higher value n' during the data acquisition procedure, which means that the end time of the acquisition process does not necessarily have to be known a priori. Secondly, we can adjust the sampling probability p over time. If the set of dominant frequencies does not vary, which is the case when human motion is considered that follows regular patterns, then by Eq. (8), p can be reduced in order $\frac{\log n}{n}$ as time goes by, without an accompanying reduction of recovery quality. This is remarkable since an acquisition of a signal over a long time period with the proposed algorithm therefore requires relatively less effort than the acquisition of a short signal.

4 Experimental Evaluation of the Proposed Approach

Two aspects need to be regarded here in order to examine the applicability of the proposed sampling scheme. On the one hand, the data recovered from a random signal acquisition has to serve its purpose in its subsequent processing steps. On the other hand, the acquisition of the partial data on the mobile device must indeed consume less energy than the conventional method, ideally proven with actual measurements. In this paper, we confine ourselves to an evaluation of the former, leaving the latter for future research. In particular, we assess how valuable recovered sensor data is for the task of motion recognition.

The dataset used for the evaluation was recorded by a smartphone which was attached to a rocker board. We recorded the orientation of the board in three different directions via acceleration sensors, as they are often a primary source for motion data on mobile devices. In this way, the orientation of the rocker board could be measured unambiguously. During the experiment, a person was standing on the board and moved the board with his/her feet, while the smartphone recorded the orientation of the board, see figure 2. This way, a total of 1540 orientation signals have been acquired with six test subjects. The participating persons always performed one of three different movements. (i) The person either moved the rocker board back and forth, (ii) left and right or (iii) the person tried to balance on the center of the board.

In order to evaluate the presented approach, we simulate an acquisition of these signals with the proposed algorithm and with conventional downsampling, both with equal average sampling rates. To recover the signal from the samples which were acquired by the presented algorithm, we apply the NESTA method [14]. As a simple tool to classify the acquired data, a principal component analysis (PCA) is used.

We use 90% of the data for training the classifier and 10% as test data. The data acquired with (a) Compressive Sensing and (b) downsampling are projected onto the first 10 principle components and the euclidean distance to the trained centers is used for classification.

Fig. 2. The rockerboard in usage at the experiment

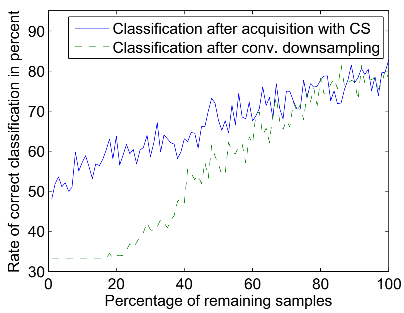

Fig. 3. The results of the classification for different sampling rates via Compressive Sensing and the conventional approach

Figure 3 shows the classification results of the conducted PCA. It can be seen that an acquisition of a signal with the proposed sampling algorithm leads to higher rates of correct classification, especially if the sampling rate decreases. We assume that more sophisticated classifiers amplify this promising effect.

5 Conclusion

For several mobile scenarios, sensor data capturing and data processing can be separated. For these applications, Compressive Sensing can significantly reduce the sampling rate on the mobile device. We plan, to investigate as part of our future work, to incorporate other factors influencing battery consumption, for example when and how often to communicate the acquired data for remote offline processing. Our experimental results show that a higher rate of correct classification is possible in motion recognition, if the signal is acquired with the presented approach.

We furthermore proposed a general sampling algorithm for time-dependent signals, which is not restricted to motion recognition applications. One of its interesting properties is that it even allows a reduction of the sampling rate, if the length of the data acquisition is spontaneously increased. As a downside however, the presented sampling algorithm requires the mobile device to make certain computations before each acquired sample. In the light of the promising experimental results, it therefore remains an interesting task to investigate the actual reduction in energy consumption for the presented approach.

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