V2V Communications in Automotive Multi-sensor Multi-target Tracking

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***Abstract*—Today's automotive sensor systems for in-vehicle based target tracking, i.e. radar, lidar, camera, are limited to a field of view which is restricted by distance, angle and line-of-sight. Future driver assistance systems such as predictive collision avoidance or situation-aware adaptive cruise control require a more complete and accurate situation awareness in order to detect hazardous and inefficient situations in time.**

Therefore, we introduce multi-target tracking including Vehicle-2-Vehicle communications as a complementing sensor for future driver assistance systems. The paper presents first simulation results of our algorithm which show promising outcomes.

I. Introduction

Vehicle-2-Vehicle (V2V) communications allows the exchange of information between vehicles (inter-vehicle communications) and between vehicles and infrastructure (infrastructure-vehicle communications). This information can then be used for instance in driver assistance systems to improve safety, efficiency and comfort of driving. Novel applications are for instance traffic jam warning, cooperative collision detection or cooperative merging assistance [1].

On the other hand V2V communications allow the enhancement of already existing driver assistance systems. Examples are cooperative navigation or cooperative adaptive cruise control. Conventional adaptive cruise control works similar to cruise control with the difference that the speed of the ego vehicle is decreased if another vehicle is in the headway [2]. The detection and ranging of other vehicles usually is based on radar, lidar or even optical camera sensors (in the following referred to as *autonomous detection and ranging sensors*).

As long as the preceding vehicle is located within the detection zone and no obstacles such as other vehicles, buildings, guard rails, etc. obstruct the line-of-sight, target detection and ranging of autonomous sensors is subject to conventional signal propagation errors. These errors emerge due to:

- environmental impact: optical systems (e.g. lidar, camera) show significant deterioration in fog, rain or snow causing noisy measurements
- unintended reflections and scattering on guard rails, buildings, secondary vehicles, etc. causing the occurrence of "ghost vehicles"
- poor angular resolution of automotive radar sensors causing undetection of vehicles [3]

Some of these errors can be mitigated by using dynamic state estimation and sensor fusion (e.g. camera & radar) [4], e.g. by Kalman filters or particle filters. But the unavailability that arises if the target vehicle is not located in the detection zone or shaded by obstacles is not mitigatable likewise. This problem can be traced back to the fact that autonomous detection and ranging sensors rest upon the direct reflection of microwave, laser or optical signals and thus are limited to the line-of-sight zone.

We presented the fusion of autonomous relative position measurements and position information provided by V2V communications in [5] as a possible solution. While this approach is limited to the relative distance estimation of a single target vehicle, this paper will point out the applicability on a real traffic situation with multiple vehicles in the vicinity.

This is especially important for future predictive driver assistance systems which have to observe the situation with higher integrity in order to base the prediction on a sufficiently complete and accurate situation model. This approach will pave the way for situation-aware driver assistance systems that enable a multitude of novel applications and improvements of already existing applications [6].

Section II gives an overview on multi-target tracking. Section III introduces the integration of V2V communications to multi-target tracking. Section IV describes the implementation of the multi-sensor multi-target tracking based on a particle filter. A conclusion is given in section V.

II. Multi-target tracking

A. Target Tracking

Target tracking means detection and ranging of relevant objects over time [4]. The only available information related to the target are noisy and incomplete sensor measurements including so-called ghost vehicles. Thus, the objective of target tracking is the dynamic state estimation of a target based on this set of noisy and incomplete measurements. For that purpose, dynamic state estimators make use of the temporal correlation of the measurements to mitigate the measurement noise in the state estimation. In order to quantify this correlation, a movement model of the target and an observation model of the sensor are essential.

If only the most recent hidden state is inferred given past measurements, this is called *filtering*. Prominent algorithms, such as Kalman filter or particle filter, exploit Bayesian theory for the state estimation. An overview on Bayesian filtering can be found in [7].

In principle, the dynamic state estimator filters the noisy sensor measurements $z^{1:k}$ over the time span 1 to k and adequately infers the posterior distribution of the state space $p(x^k|z^{1:k})$ at time k which will include at least the relative position of the target vehicle. According to [7] filtering can be seen as an iterative *prediction-correction process* comprising the two recursive steps: *prediction* and *update* (see fig. 1). The prediction step of the dynamic state estimator is defined by:

$$p(\mathrm{x}^k|\mathrm{z}^{1:k-1}) = \int p(\mathrm{x}^k|\mathrm{x}^{k-1})p(\mathrm{x}^{k-1}|\mathrm{z}^{1:k-1})d\mathrm{x}^{k-1} \quad (1)$$

The update step is defined by:

$$p(\mathrm{x}^k|\mathrm{z}^{1:k}) = \frac{p(\mathrm{z}^k|\mathrm{x}^k)p(\mathrm{x}^k|\mathrm{z}^{1:k-1})}{p(\mathrm{z}^k|\mathrm{z}^{1:k-1})} \quad (2)$$

To solve the equations, we prefer particle filtering over other filter techniques such as Kalman filter because it allows the usage of non-Gaussian measurement and movement noise and non-linear measurement and movement models [8], [9]. Especially for complex non-linear driver behavior modeling and observation models this is an essential requirement.

The key idea of particle filters is to represent the posterior distribution by a set of discrete samples, so called *particles*. These particles are used in a sequential Monte Carlo method for Bayesian inference to predict and update the estimated state of the target based on the observations. As derived from the general rule for Monte Carlo sampling, the accuracy of the state estimation strongly depends on the number of particles used.

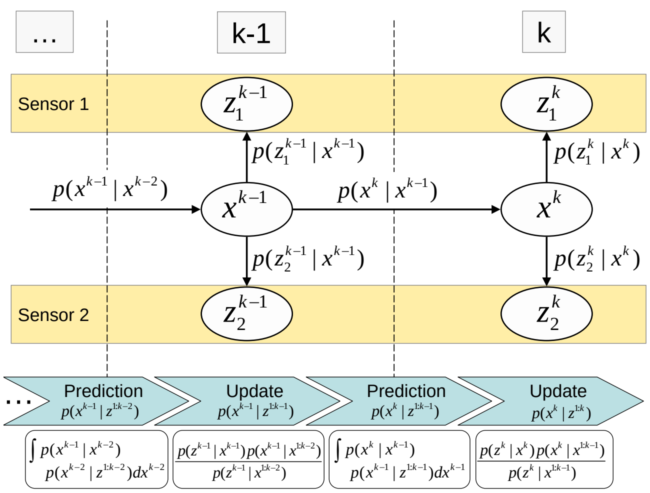


Fig. 1. **Predication and Update** on the state space x^k with two sensors providing sequential observations $\{z_1^k, z_2^k\}$ at time step k

B. Tracking Multiple Targets

In application environments, such as road traffic, usually multiple moving targets with unknown and changing quantity are present. Measurements are anonymous and cannot directly be associated with the targets. Thus, dynamic state estimation becomes more complex due to the additional problem of associating measurements to targets, sometimes also called *report-to-track*, *object-data* or *state-observation* association. The problem even gets more complicated if there is no one-to-one relation between measurement (N) and target (T). Hence, the following constellations may occur:

- 1:1: a single measurement is caused by exactly one target and this is the only measurement caused by this target
- $N : 1$ with $N > 1$: a single target can cause more than one measurement (e.g. due to signal scattering)
- $1 : T$ with $T > 1$: a single measurement can be related to more than one target (e.g. no target separation due to limited angular resolution [3])
- $N : T$ with $N > 1, T > 1$: T targets can cause N measurements

For the tracking of multiple targets given noisy and incomplete measurements the last constellation is of main importance. To solve the estimation and association problem different approaches have been used.

a) Single Hypothesis: One approach for multi-target tracking is to consider each target separately from others and track it with a separate filter. Each filter thus handles a single hypothesis, i.e. the most probable hypothesis given the observations. Single hypothesis are used for example in [10], [11], [12]. These methods keep only one hypothesis of the tracking result which has the most probable posterior distribution based on current and previous observations. Thus they may fail with background clutter, occlusions and multi-object confusions.

b) Multiple Hypotheses: Instead of making the decision on the most probable posterior distribution, i.e. target constellation, at each time step, the multiple hypotheses approach takes into account several possible target constellations and infers these hypotheses so that the uncertainty in the correct target constellation can be reduced on the arrival of subsequent observations. Multiple hypotheses methods are more robust because the tracking result corresponds to the state sequence which maximizes the joint state-observation probability.

The multi-target tracking problem has been traditionally addressed with techniques such as *multiple hypotheses tracking (MHT)* [13] and *joint probabilistic data association (JPDA)* [14] which is a special case of MHT. Both techniques work by translating a measurement into a set of targets by thresholding. The detections are then either associated with existing targets, used to create new targets, or deemed false alarms.

In the work of Orton and Fitzgerald in [15] which was based on [10] and [16] the authors represented each hypothesis subdivided in a set of n partitions. The objects to track are seen as elements of a random set, i.e. a set of random variables, for which the cardinality is itself a random variable. This is strongly related to the theory of *finite set statistics (FISST)* [17] and *joint multi-target probability density (JMPD)* [18]. This can be seen as purely Bayesian perspective. Measurement-to-target associations are not done explicitly e.g. by thresholding; the association is performed implicit within the Bayesian framework.

C. Multi-Vehicle Tracking

The multi-target tracking with multiple hypotheses allows the dynamic tracking of an unknown and changing number of vehicles which cause a set of noisy and incomplete measurements. These measurement are the only evidence which can de facto be exploited besides a priori knowledge such as movement models and sensor characteristics. Based on this information the tracking algorithm has to guarantee a high grade of completeness and accuracy. This is especially important in the application area of vehicle detection in future situation-aware driver assistance systems which, instead of performing actions directly on the occurrence of evidence, attempt to estimate the causative situation, i.e. the causes of evidence. Although we inspect merely vehicle tracking in this paper, we do not limit the algorithm to this application area. Other examples, where multi-target tracking can be applied, are environmental phenomena, such as wet, icy or oily road conditions, traffic phenomena, such as traffic jams or traffic hold-ups, or any other situational information.

As an example, the detection of a new vehicle which comes into the field of view of the ego vehicle's radar is depicted in fig. 2. The simulation is based on the implementation described in section IV.

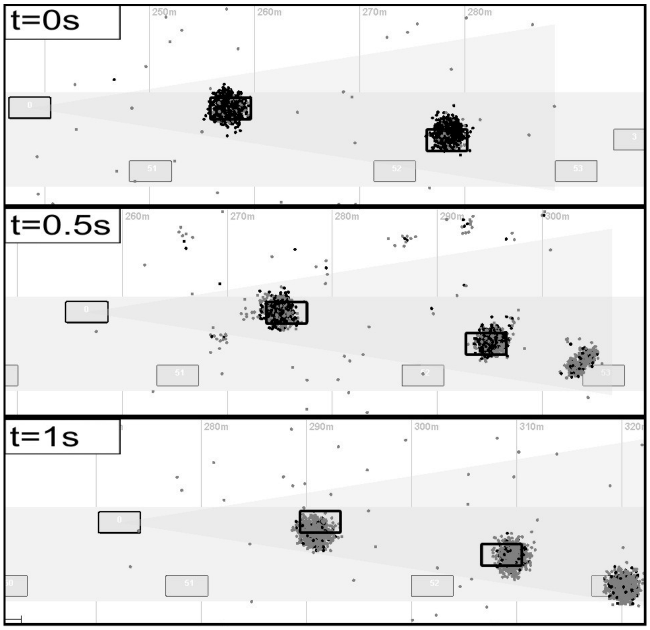


Fig. 2. Target Vehicle Detection with standalone radar:
t=0s: The particle filter tracks 2 vehicles which are in the field of view of the autonomous detection and ranging sensor. The number of tracked vehicles is 2 for the majority of hypotheses (particles representing hypotheses with 2 partitions are filled with black color)
t=0.5s: The particle filter tracks an additional vehicle on the right most lane. A subset of the hypotheses includes already 3 vehicles (particles representing hypotheses with 3 partitions are filled with gray color)
t=1s: The particle filter tracks 3 vehicles. Almost all hypotheses have 3 tracked vehicles (particles representing hypotheses with 3 partitions are filled with gray color)

III. Tracking Complementation by V2V communication

The multi-target tracking algorithm provides a highly accurate detection and ranging mechanism to track vehicles which are in the *field of view (FOV)* of the autonomous sensor system (see fig. 2). But the tracking is restricted to the FOV of the autonomous sensor system which is strongly limited in distance and angle, and obstacles that block the line-of-sight. These problems can be overcome by the integration of an additional sensor system that is not subject to these limitations. Thus, we propose to complement target tracking by the integration of V2V communications as a virtual sensor.

One of the basic functionalities of future V2V communication systems will be periodic beaconing. These beacons include among other status information the current position, speed and heading of the vehicle. On the one hand this is used for application-related purposes for safety, efficiency and comfort and, on the other hand, for route maintenance on network layer [1]. Beacon messages distributed via V2V communications are usually propagated in an omnidirectional manner and thus are not affected by angular FOV limitations. Furthermore the propagation area is much larger than for autonomous sensors; and may even be extended by multi-hop communications if required. Last, obstacles between the ego vehicle and target vehicle have less bearing than for autonomous sensor systems.

Complementing the multi-target tracking described in section II with V2V communication hence overcomes the limitation given by the autonomous sensor system and thus provide higher accuracy, better reliability and increased robustness against sensor failures. Based on the fact that this cooperative detection and ranging strongly depends on the penetration rate of V2V communication equipment, we consider cooperative detection and ranging not as substitute for autonomous methods but as a promising complementation which will unfold its potential with increasing penetration rate.

In order to detect and range target vehicles the V2V message has to include information regarding the position of the target vehicle. This information can for instance be obtained by the global navigation satellite system (GNSS). There are different concepts to express position relevant data obtained by GNSS:

- **Absolute position based relative positioning** by differencing of two absolute positions. This method may be influenced by the whole set of GNSS measurement errors (satellite clock offset, satellite orbit dislocation, ionospheric and tropospheric refraction, receiver clock offset and multipath propagation).
- **Code based relative positioning** uses a *Time Difference of Arrival (TDoA)* method with several simultaneous measurements of different satellites on a code basis. Ego vehicle and target vehicle have to use identical satellites at the same time. Depending on the algorithm the following errors can be eliminated:
  - Single differencing between receivers eliminates pseudo-range errors emerging from satellite clock bias, satellite orbit dislocation and ionospheric and tropospheric refraction. The different types of errors have a high correlation when signals emitted from the same satellite at the same time have a similar propagation path which is valid within short distances between ego vehicle and target vehicle as it is considered in this paper.
  - Double differencing between satellites additionally eliminates errors emerging from receiver clock offsets.
- **Carrier based relative positioning** uses TDoA on a carrier basis. Besides single and double differencing, triple differencing between epochs has to be considered in order to quantify integer cycle ambiguity.

Depending of the type of application and its requirements a suitable approach for the position relevant data has to be chosen and the respective messages have to be defined. For our initial simulations we used absolute position based relative positioning.

IV. Implementation

As a proof-of-concept we implemented the multi-sensor multi-target particle filter (mainly based on the work of Kreucher et al. [18]) and coupled it to our existing traffic simulation environment. The core components are detailed in the following subsections.

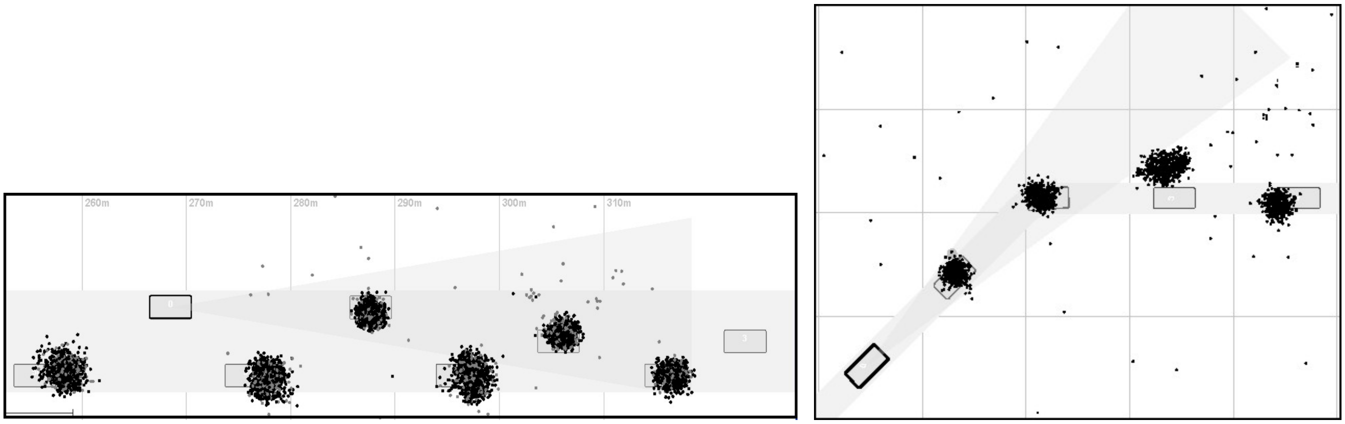
A. Particle Filter

The particle filter implementation is based on the *sample importance resampling (SIR)* algorithm which is a special case of the *sequential importance sampling (SIS)* algorithm [9]. The posterior probability distribution is represented by a set of 1000 weighted particles each of which forms an independent hypothesis of the state at a given time, i.e. a representation of a possible situation. Updates are performed with 5 Hz.

In principle three main components form our particle filter implementation for multi-sensor multi-target tracking which are described in the following.

1) Hidden State Space (Situation Model): The hidden state space is based on partitioned hypotheses. Each partition x encloses the latitudinal distance x_{lat}, the longitudinal distance x_{lon}, the movement direction x_h and the movement speed x_v of a single tracked target.

$$\mathrm{x} = [x_{lat}, x_{lon}, x_h, x_v] \tag{3}$$



(a) Straight 3-lane road scenario: The majority of hypotheses comprise 6 tracked vehicles (hypotheses with 6 partitions are filled with black color, other hypotheses are depicted gray)

(b) Winding single-lane road scenario

Fig. 3. Multi-sensor Multi-target tracking: Vehicles located within the FOV of the autonomous sensor system and V2V communication area show a condensed estimation of the target vehicles' position. Vehicles located merely in the V2V communication area show a more spread position estimation but, nevertheless, have a mean of the distribution with sufficient accuracy.

Additionally, the number of partitions T, i.e. the number of tracked targets, completes the notation of the hidden state S at time k:

$$\mathrm{S}^k = \{\mathrm{X}^k, \mathrm{T}^k\} \qquad with \;\; \mathrm{X}^k = \{\mathrm{x}_1^k, \ldots, \mathrm{x}_T^k\} \tag{4}$$

Thus, the number of partitions appears as an additional discrete random state variable that defines the dimensionality of the hypothesis. Consequently, the state space has different dimensionality for different values of T which can change dynamically. This differs from traditional particle filter implementations with a static notation of hypotheses.

2) State Transition Model: The state transition model first performs a transition of the random variable T. This allows the detection of new targets and the gating of outdated targets. In our implementation T is incremented/decremented by 1 both with a probability of 10% which allows a fast enough acquisition of new targets and gating of outdated targets.

In the next step the partitions are adjusted to the new value of T. A new partition is created if T have been incremented or an existing randomly chosen partition is removed if T has been decremented. If a new partition is created, a random sample according to the prior distribution is set as initial instantiation. The initial instantiation during runtime is similar to the initial instantiation at system startup but displaces the initial target positions more towards the edges of the sensor detection zone as this is more probable during runtime.

Subsequently, for all partitions a point rotation of x_{lat} and x_{lon} according to the change in the ego vehicle heading is performed. Additionally a point translation is performed which is subject to Gaussian noise. The point translation mainly depends on the speed and movement direction of the own and the target vehicle. In order to gain more precise movement models additional information, such as speed and heading of the target vehicle which is also included in its V2V beacons, can be exploited dynamically in the point translation. We did not model the dependencies of the movement related to the movement of other target vehicles. So there is no synchronization between the movement models up to now.

3) Observation Model: For time k a set of measurements Z^k is provided by n sensors:

$$Z^k = \{\mathrm{z}_{1,1}^k, \ldots, \mathrm{z}_{1,d_1^k}^k, d_1^k\} \cup \ldots \cup \{\mathrm{z}_{n,1}^k, \ldots, \mathrm{z}_{n,d_n^k}^k, d_n^k\} \tag{5}$$

d_i^k is the dimension of the measurement set provided by sensor i at time k.

In our current implementation Z^k consists of measurements from two different sensors namely the radar system and the complementing V2V communication but may be easily extended in the future.

In accordance to equation 2 the update step for the multi-sensor multi-target tracking is defined by:

$$p(\mathrm{X}^k, \mathrm{T}^k|\mathrm{Z}^{1:k}) = \frac{p(\mathrm{Z}^k|\mathrm{X}^k, \mathrm{T}^{\mathrm{k}})p(\mathrm{X}^k|\mathrm{Z}^{1:k-1})}{p(\mathrm{Z}^k|\mathrm{Z}^{1:\mathrm{k}-1})} \tag{6}$$

Measurements come into the play in the update step of the dynamic state estimation. For the update step the measurement likelihood $p(\mathrm{Z}^k|\mathrm{X}^k, \mathrm{T}^k)$ has to be determined. Informally, the question is *"How likely are the measurements Z^k given a certain target constellation?"* Thereby we rely on the strict causal relation from the target to the measurements and thus do not establish any direct association of a single measurement to a specific target but merely estimate the likelihood of a measurement set given a specific target constellation.

The measurement sets of different sensors are independent given the target state, i.e.

$$p(\mathrm{z}_{1,1}^k, \ldots, \mathrm{z}_{1,d_1^k}^k, d_1^k, \ldots, \mathrm{z}_{n,1}^k, \ldots, \mathrm{z}_{n,d_n^k}^k, d_n^k|\mathrm{X}^k, \mathrm{T}^k) = \prod_{i=1}^{n} p(\mathrm{z}_{i,1}^k, \ldots, \mathrm{z}_{i,d_i^k}^k, d_i^k|\mathrm{X}^k, \mathrm{T}^k) \tag{7}$$

Thus we evaluate each sensor measurement set independently by evaluating $p(\mathrm{z}_{i,1}^k, \ldots, \mathrm{z}_{i,d_i^k}^k, d_i^k|\mathrm{X}^k, \mathrm{T}^k)$. For the evaluation we iterate over the individual measurements and calculate the probability that this measurement is caused by one (or more) of the targets. A certain minimum likelihood assert the occurrence of false positives due to clutter, reflections, etc.

A larger target set would get a higher weight because measurements become more likely although this does not reflect the real situation. That is why we additionally compare the number of targets T^k with the number of caused measurements d_i^k. For this we have to consider that there does not need to be a 1:1 relation of targets and measurements as described in section II. Furthermore, the surveillance areas of our sensors are not concordant and, hence, this comparison should only take the targets into account that are within the surveillance area of the respective sensor. Thus, for the radar sensor only the targets that are within its FOV and not the additional targets detected by the V2V communications are taken into account.

B. Simulation Environment

In order to validate our concepts we designed a simulation environment that allows the simulation of the multi-sensor multi-target tracking in reproducible traffic situations. Therefore we implemented a radar sensor with an opening angle of 9° and a maximum range of 50 meters. The radar sensor incorporates various measurement errors. Vehicles that are not detected mainly arise due to the limited FOV and shading by obstacles. Wrongly detected vehicles occur due to reflection on obstacles (such as guard rails, buildings, or roadside planting). For the quality of each measurement we used a 0-mean Gaussian measurement noise with $\sigma = 1m$.

The cooperative detection and ranging was based on absolute position based relative positioning with a constant 0-mean Gaussian measurement noise with $\sigma = 5m$. Transmission errors were not modeled adequately because a small number of vehicles and a high beaconing rate (10 Hz) of position relevant information was used and thus sporadic message losses can be neglected for the overall observation.

The functioning of our implementation is shown in fig. 3(a) on a straight multi-lane road with multiple vehicles moving in the same direction and in fig. 3(b) on a winding road with 4 vehicles in the headway. The figures show a high detection rate for all vehicles in the vicinity and a high accuracy for the position estimation of the vehicles located in the FOV of both sensors, the autonomous and the cooperative detection and ranging sensor. The vehicles that are merely in the FOV of the cooperative sensor have less accurate position estimations (particles are more spread) but the mean of the distribution is highly accurate in most cases and thus can serve as a very good starting point for the situation assessment of future driver assistance systems.

V. Conclusion

In this paper we identified the potential of complementing existing autonomous detection and ranging systems (e.g. radar, lidar, camera) with V2V communications. In particular this paper pointed out the applicability of an additional sensor in multi-target tracking for future situation-aware driver assistance systems. The results as depicted in fig. 2, 3(a) and 3(b) of our multi-sensor multi-target tracking implementation based on a particle filter show promising outcomes.

The more complete and more accurate relative position information of the vehicles in the vicinity (extending the FOV of autonomous sensors) can be used as a basis for future situation-aware driver assistance in order to predict hazardous or inefficient situations in time. The already mentioned cooperative adaptive cruise control as an example can hence react on vehicles abruptly changing the lane or driving maneuvers of vehicles that cause the preceding vehicle to slow down fast (e.g. because of an upcoming traffic jam).

This information can not only be used by a cooperative adaptive cruise control but also to warn the driver of a potential risk of a collision, detect traffic jams or enable autonomous driving. The multi-sensor multi-target tracking can thus be seen as a basic functionality which paves the way for a multitude of driver assistance systems.

References

[1] "Car-2-Car Communication Consortium," http://www.car-2-car.org.

[2] ISO 15622, *Transport information and control systems - Adaptive Cruise Control systems - Performance requirements and test procedures* , Std., 2002.

[3] R. Rasshofer and K. Gresser, "Automotive radar and lidar systems for next generation driver assistance functions," in *Advances in Radio Science*, 2005.

[4] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Artech House Radar Library, 1999.

[5] M. Röckl, J. Gačnik, J. Schomerus, T. Strang, and M. Kranz, "Sensing the environment for future driver assistance combining autonomous and cooperative appliances," in *4th Workshop on Vehicle-to-Vehicle Communications (V2VCOM)*, 2008.

[6] M. Röckl, P. Robertson, K. Frank, and T. Strang, "An architecture for situation-aware driver assistance systems," in *IEEE 65th Vehicular Technology Conference - Spring 2007*, I. VTS, Ed., April 2007. [Online]. Available: http://elib.dlr.de/45042

[7] Z. Chen, "Bayesian filtering: From Kalman filters to particle filters, and beyond," Adaptive Syst. Lab., McMaster Univ, Tech. Rep., 2003.

[8] K. Wendlandt, M. Khider, M. Angermann, and P. Robertson, "Continuous location and direction estimation with multiple sensors using particle filtering," in *MFI 2006*. IEEE, September 2006. [Online]. Available: http://elib.dlr.de/44240

[9] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, and P.-J. Nordlund, "Particle filters for positioning, navigation and tracking," in *IEEE Transactions on Signal Processing*, vol. 50, no. 2, 2002.

[10] J. MacCormick and A. Blake, "A probabilistic exclusion principle for tracking multiple objects," *International Journal of Computer Vision*, vol. 39, no. 1, pp. 57–71, 2000.

[11] H. Tao, H. S. Sawhney, and R. Kumar, "A sampling algorithm for tracking multiple objects," in *Workshop on Vision Algorithms*, 1999, pp. 53–68.

[12] C. Hue, J. L. Cadre, and P. Perez, "Tracking multiple objects with particle filtering," IRISA, oct. 2000., Tech. Rep., Oct. 2000.

[13] D. Reid, "An algorithm for tracking multiple targets," *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.

[14] T. Fortmann, Y. Bar-Shalom, and M. Scheffe, "Sonar tracking of multiple targets using joint probabilistic data association," *IEEE Journal of Oceanic Engineering*, vol. 8, no. 3, pp. 173–184, 1983.

[15] M. Orton and W. Fitzgerald, "A Bayesian approach to tracking multiple targets using sensor arrays and particle filters," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 216–223, 2002.

[16] J. MacCormick and M. Isard, "Partitioned sampling, articulated objects, and interface-quality hand tracking," in *ECCV (2)*, 2000, pp. 3–19.

[17] R. Mahler, "An introduction to multisource-multitarget statistics and its applications," Lockheed Martin Technical Monograph, Tech. Rep., 2000.

[18] C. Kreucher, K. Kastella, and A. Hero, "Multitarget tracking using the joint multitarget probability density," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 4, pp. 1396–1414, 2005.